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The Russian economy has evolved into a hybrid form, a partially monetized quasi-market system that has been called the *virtual economy*. In the virtual economy, barter and non-monetary transactions play a key role in transferring value from productive activities to the natural monopolies and loss-making sectors of the economy. The project aims at providing a consistent and multi-facet exposition of main features of the Russian virtual economy, and explains why it is different from other transition economies. In particular, the endogenous appearance of money substitutes and wide-spread use of non-monetary transactions are explained as a result of the optimizing behavior of agents in the economy, given the inherited distorted structure, the existence of large natural monopolies, the unproductive sector organized into informal networks, and the government policies aimed at the support of this sector. We highlight the typical distortions of the price system, and provide an explanation of the observed dynamics of the volume of money substitutes in circulation after August 1998 crisis. We show that the Russian virtual economy can be in one of two steady states with approximately the same price systems but essentially different volumes of money substitutes in circulation. Transition into the state with the smaller volume of money substitutes in circulation can be caused by an increase of the number of active producers resulting from some external shock. We conjecture that should the productive business activity be suppressed, then the economy can swiftly return to the former state with the large volume of money substitutes in circulation.

Keywords: Russia, money substitutes, search models

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CONTENTS

1. INTRODUCTION

Our goal is to explain the basic properties of Russia's economy, especially wide-spread usage of money substitutes and price distortions, as resulting from the structure inherited from the Soviet Union and the government policies. It is also necessary to examine the mechanism of the transfer of value from more productive agents to less productive ones.

Starting with Karpov's report (1997) (see Kuznetz (1998)) and papers of Gaddy and Ickes (1998), it has become a widely spread perception of the Russian economy as the *virtual economy* with a number of special features different both from a market economy and a command economy (see Guriev and Pospelov (1998), Ericson (1998), Gaddy and Ickes (1999), Guriev and Ickes (1999), Ericson and Ickes (1999, 2000), Marin (2000), Guriev and Kvassov (2000), Guriev and Makarov (2000) and the bibliography there). Certainly, any transition economy differs from the above types of economies, but a word "transition" presupposes a passing stage of a transformation from a command economy into a normal market one whereas the Russian Economy enjoys several stable abnormal properties, which do not exhibit a tendency to fade out. This means that the Russian Economy may be in a new steady state, which rises the necessity of constructing appropriate macroeconomic models.

One of the key features of the Russian Economy is the wide-spread use of barter and money substitutes (see e.g. empirical studies Aukutsionek (1994, 1998), Commander and Mumssen (1998), Dolgopiatova (1998), Guriev and Ickes (1999)), which is much more prominent than in other Transition Economies (Carlin, Schaffer and Seabright (1999)). At the same time, after August 1998 crisis, the volume of money substitutes in circulation decreased significantly, though no serious restructuring had taken place. To understand properly the nature of the problem, it is necessary to realize which exogenous factors lead to the endogenous appearance of money substitutes, and how the economy reacts to various subsidy policies.

Models which explain the *endogenous* appearance of money substitutes are necessary because they highlight the main structural problems, which lead to the reappearance of money substitutes (by the way, we know of a network of interconnected firms in Rostov region, which did not use money substitutes till August 2000 but started to use them since then; they use vekselya when it comes to settle mutual arrears in the beginning of each quarter). In particular, any model that has no role for endogenous use of means of exchange will give unduly importance to purely monetary policy mechanisms. The main message of our model is that the Russian economic system will produce sufficient amount of endogenous liquidity at any circumstances, and that it can be in two steady states, with essentially different volumes of notes in circulation but approximately the same price systems.

Besides, if one is to analyze possible future states of the economy after this or that sort of restructuring, one must model the *endogenous* dynamics of prices, which crucially depends on the properties of various means of exchange circulating in the economy. The important consequence of the peculiar structure of Russian economy is the transfer of value from the productive enterprizes to unproductive ones.

According to the contemporary Monetary Theory it does not suffice just to state the existence of money and money substitutes and list the empirical facts about their use. Also, without the modeling of properties of monies as arising endogenously, it is impossible to access future prices. Unfortunately, this very important fact is overlooked, when investors access Russian firms or investment projects: if one seriously believes in changes in the future, then one has to acknowledge the fact that the net present value of an investment project calculated under assumption of the relative stability of the existing price system is bound to drastically depart from the reality[1]. In addition, the whole price system of the economy will change should the subsidy system change.

As far as the study of Russia's economy concerned, our model is the first macro-model based on the principles of the modern Monetary Theory, which studies the interactions among the main actors of the economy. At the same time, we obtain new results for Monetary Theory per se.

The endogenous appearance of money substitutes in an economy is an old issue in Monetary Economics; we are interested in money and money substitutes as media of exchange. Walrasian equilibrium models implicitly assume an auctioneer, who observing the goods suggested for trade chooses market clearing prices. Hence, Walrasian equilibrium (WE) models have no role for a valued fiat medium of exchange, and since they always assume complete markets, all assets can be traded at given prices in any circumstances. Therefore, all monetary theories have to depart from WE.

[1] In the situation, when the government controls most of the economy, changes can take place at any moment: suppose, that the government decides that not only UES, but Gazprom as well should stop to accept and issue vekselya

However, some monetary models (like money-in-the-utility/production-function, or transaction costs models) depart only from Walrasian physical environment (agents, preferences, resources, technology, information structure), while others (like trading-post models or cash-in-advance models) depart from the equilibrium concept (rules governing interactions among agents). Finally, there is a class of models which depart both from physical environment and equilibrium concepts of WE. One fraction of this class consists of models with an absence of double coincidence of wants.

Monetary theories should not contain money as a primitive — this is Wallace's dictum for Monetary Theory. Models which assume real balances being arguments of utility or production functions or impose cash-in-advance constraints do not satisfy this dictum. The main reason is that they do not permit the assets' role in exchange to be endogenous. This role is given to the assets in a model with no double coincidence of wants. Absence-of-double-coincidence notion goes naturally with pairwise meetings of agents, therefore one theory which satisfies the dictum is a random matching model. Random meetings imply that agents cannot choose whom to meet with, therefore they have to search. Monetary search models show how fiat currency can be valued, how endogenous commodity money can arise; they can also discuss international monetary issues and address a variety of other questions in monetary economics.

Random matching monetary models with indivisible goods and money study exchange processes where, once agents meet, they exchange and part company. However, this framework is not interesting enough because prices are given exogenously. One of the ways to generalize these models is to make goods divisible, then the rate at which agents exchange can be determined by bilateral bargaining. A strategic bargaining model is due to Rubinstein and Wolinsky. This is an essentially dynamic model, however it is possible to show that the equilibrium outcome of the strategic bargaining game can be approximated by the generalized Nash bargaining solution which is inherently static, and therefore tractable.

One of the important questions addressed by monetary economics is whether the private sector should be allowed to create money. One concern, which holds for any economy, is what mechanisms could prevent a private monetary system from printing too much money. Monetary models which use random matching to represent a trading process, were introduced in Kiyotaki and Wright (1989; 1991; 1993); for subsequent developments, see Trejos and Wright (1995), Aiyagari, Wallace and Wright (1995), Shi (1997), Wallace (1997) and review Wallace (1998). Models of this type decentralize the trading frictions, abandon the Walrasian fiction and naturally generate transaction demand for money. There are several search-theoretic models incorporating *inside money*. The word *money* indicates the object which is used as a tangible medium of exchange among the agents who recognize it as an asset, and the label *inside* shows that this object is supplied by the private sector. In Cavalcanti et al. (1999), Cavalcanti and Wallace (1999 a, 1999 b), and Williamson (1999) inside money is given a role both as credit and as a tangible medium of exchange. In Cavalcanti et al. (1999), Cavalcanti and Wallace (1999 a, 1999 b), it is assumed that a fraction of the population — a banking sector — has access to a private note-issuing technology, while the rest of the economy — a non-banking sector — uses inside money as a medium of exchange. In Williamson (1999), a model with claims on banks as private money is explored. Agents can choose between investing into low or high-return projects, so there may exist welfare dominated equilibria where banks hold low-return assets. There is also a paper Burdett et al. (2001), which introduces endogenous money as a commodity that can be either consumed or stored and used as a medium of exchange. In this model, there is no role for credit because the trade, if it takes place, is always *quid pro quo*.

While the endogenous (commodity) money does not require any kind of pre commitment, since the exchange, if it takes place, is always *quid pro quo*; models with inside money need incentives of the agents to be taken into consideration. Though, originally, credit was completely ruled out of the random matching framework, it is possible to introduce some form of credit into monetary models by assuming that either people can commit to future actions, or (complete or partial) public information about trading histories is available, or both. The first paper to put partial public knowledge into the random matching setting, was the one of Kocherlakota and Wallace (1998). In Cavalcanti and Wallace (1998; 1999) and Cavalcanti, Erosa and Temzelides (1998), it is assumed that a fraction of population — a banking sector — has access to a private note-issuing technology, while the rest of the economy — a non-banking sector — uses inside money as a medium of exchange. There is a record keeping technology (clearing house) in the first sector and privacy-of-trading histories in the second sector. There is also a note redemption technology inside the banking sector which allows to discipline the amount of notes issued by the banking sector. Williamson (1999) explores a model with claims on banks as private money. Agents can choose between investing into low or high-return projects, so there may exist welfare dominated equilibria where banks hold low-return assets. Also it is shown that in case of private information, private money may be subject to lemons problems.

In addition to the media of exchange produced in the private sector, all the above models with inside and endogenous money incorporate the exogenous provision by a public sector of fiat currency usually referred to as *outside* or exogenous money. In Burdett et al. (2001), Cavalcanti and Wallace (1999 a, 1999 b), it is shown that an equilibrium can be achieved in an economy with only endogenous or inside money (respectively) in circulation. The former paper also shows that if the supply of exogenous money is sufficiently small, both types of money may coexist. Cavalcanti and Wallace (1999 a, 1999 b) consider implementable allocations that arise with inside and outside money separately; they do not examine coexistence of both kinds of money. In Cavalcanti et al. (1999), coexistence of private and government money is studied only for the case of a discount factor close to one, but no analytical results for endogenous variables are obtained.

In order to explain the endogenous appearance and circulation of money substitutes in the economy, we take the monetary search approach. Agents are placed in a standard environment (see Kiyotaki and Wright (1993)), in which different people have different preferences over a large number of differentiated goods. Our model combines more essential features than models in op.cit. The closest to our paper are papers Cavalcanti and Wallace (1998, 1999), Burdett et al (1998) and Cavalcanti et al (1998). Cavalcanti and Wallace (1998, 1999) consider implementable allocations which arise with inside and outside money separately, they do not examine coexistence of both kinds of monies. Burdett et al (1998) consider only commodity money as inside money, so they do not have to take into account incentive compatibility constraints etc, since the trade if it takes place is always quid pro quo. In Cavalcanti et al (1998), there is a finite number of consumption goods and individuals. Agents of type i can consume good i and produce good $i+1$. A banking sector is a real banking sector with clearing house, reserve keeping and possibility of being dissoluted if notes redeemed exceeds reserve balance. Coexistence of private and government money is studied only for the case of discount factor close to 1, but neither analytical results for endogenous variables are obtained nor the case of agents of different levels of ineffectiveness is studied (the last remark concerns other papers as well). Notice that all the models above describe the reality much simpler than Russia's economy, and cannot be directly applied to the study of the latter. Even the simpler two-sector model, which we constructed two years ago S. I. Boyarchenko, and S. Z. Levendorskiĭ (2000, 2001), lead to results which were new for the search-theoretical approach (for the detailed discussion, see op.cit.) We modeled the virtual economy as consisting of two sectors

• a sector of more productive new or restructured and privatized firms acting as individually optimizing, or private, agents, and

• a sector of old, mainly non-restructured and less productive firms, which can collude and agree on the rules for issuing and redemption of notes.

Thus, the sector of old enterprizes acts as a unique omnipresent agent. This agent redeems the notes it issues always because this is the optimizing behavior. Since the agent is omnipresent and does not defect on its obligations, it is optimal to accept its notes by everybody on conditions this agent imposes (the optimality is proved and not postulated).[2]

A note may circulate among productive agents; eventually, it is redeemed by an agent from the note-issuing sector. Since less productive agents collude, they agree on conditions for note-issuing and redemption as well as on the amount they produce in a round of trade for each other. These conditions and amount are chosen to maximize the welfare of colluding agents, given the optimizing behavior of private agents. The acceptance of a medium of exchange is decided optimally by each agent of the productive and non-productive sectors, in each trade, and the interaction of all these optimizing actions results in endogenously determined trading strategies, prices, the amount of notes in circulation and the distribution of money between private and colluding agents.[3]

We assume that agents are heterogeneous both in consumption and production, and we specify their preferences and production opportunities so that in no meeting, there is a double coincidence of wants. This naturally generates the demand for media of exchange.[4] In the previous model, we captured several interesting features of the virtual economy but the model was not quite appropriate for modeling an incentive to restructure because among the colluding agents, it did not distinguish between unproductive enterprizes and the value-adding firms in the energy sector.

In the real counterpart of our model economy, notes of such large firms as Gazprom and UES are accepted because these firms are large and omnipresent, and notes of smaller firms can circulate due to the existence of

[2] As Gaddy and Ickes (1998 a) point out, these notes (IOU's) typically circulate among chains of enterprizes short of cash and are eventually redeemed for gas, electricity etc. by some of them. The use of money substitutes is also facilitated by wide-spread stable business networks and relations, which help to organize barter (Here the word "barter" is used in the Russian sense: non-monetary transaction) chains that can use IOU's of smaller agents. For empirical study of transactions conducted using non-monetary methods and instruments, see Guriev and Ickes (1999), Carlin, Schaffer and Seabright (1999), Guriev and Kvassov (2000), Guriev and Makarov (2000) and the bibliography there.
[3] In each steady state, the strategies, prices etc. are uniquely determined
[4] The last conclusion remains valid in the more general variant of the model, where the double coincidence of wants is possible but not in any pair-wise meeting, and hence there is the genuine barter

well-established connections and barter chains, which play the role of the clearing house in search-theoretical models with the banking sector. The redemption of IOU's for goods is wide-spread in the Russian Economy, as was mentioned above. So far, our assumptions agree with the reality, except for the fact that the real old sector consists of two parts with essentially different production properties — a value adding energy part (Gazprom, UES and Oil companies; we include MPS as well but keep the name energy sector for simplicity), and low/negative value adding (loss making) old manufacturing part. Also, there was no government in our model, with either endogenous or exogenous behavior.

It is an environment with more sectors and the explicit role for the government, which we model and study now. We consider an economy with three sectors:

• a sector of new or restructured enterprizes, which act as private agents;

• a value adding energy sector (a single actor), which issue notes (money substitutes);

• a sector of old (mainly loss-making[5]) non-restructured enterprizes connected by informal networks; the latter allow the sector to choose the socially optimal level of production in trades inside the sector.

Thus, private agents are the same in our previous and current models, but colluding agents in the former represented the union of the second and third sector of the latter. The notes of the energy sector are universally acceptable as a result of optimizing behavior of all the agents of the economy (see the discussion about the two-sector model above). Private agents have no means to collude, so if such an agent issues a note, no other agent of this type is under an obligation to redeem the note. Hence each such note will be redeemed with zero probability, and in no symmetric equilibrium is it optimal for anyone to accept a note issued by a private agent. Thus we may assume that notes of private agents do not circulate at all. The energy sector designs note-issuing rules and amount of good produced in single-coincidence meetings with other agents so as to maximize the welfare of the sector. We assume that even though agents of the unproductive sector are connected by informal networks and have means to collude, they do not reveal their trading histories to the outsiders. Therefore, colluding agents are too unreliable for the outsiders so that the notes issued by such

[5] at the equilibrium prices in the sector of private agents and in trades between the energy sector and the private agents

agents are not accepted by agents of the other sectors. The agents in the productive sector are outsiders for the informal networks, and hence cannot rely on a note being redeemed. Members of a network do not defect on mutual obligations due to an additional shadow income which they obtain as members; our model does not model this fine structure of the economy.[6] We take this feature as given. We also postulate, that the energy sector does not accept the media of exchange except for the official money and notes it issues, in order not to suffer the verification cost. Among its choice variables, there is the supply of notes, and hence it can increase the supply if it is necessary.

Inside the unproductive sector, the wide-spread of informal networks results in the production-exchange policy equivalent to the policy of a central planner of the sector. Means of exchange inside the sector can be different such as notes issued by colluding agents and goods.[7] It should be noted, however, that from the point of view of monetary theory, the goods which are acquired not for the consumption but for the future exchange must be regarded not as barter goods but as a medium of exchange, called endogenous money. Only goods exchanged for consumption should be regarded as genuine barter. It is well documented that in Russia, most of the goods acquired by "barter" serve as media of exchange. This allows us to simplify the problem by eliminating barter: even if we introduce the genuine barter in our model (which is possible), it will lead to complication of the formulas but not to significant changes in our conclusions.

To sum up: we model the situation, where many essential patterns of the behavior of the large part of the economy are given exogenously, and

[6] Here is an example. The second author monitors (to rather limited extent) a network in Rostov region. The main part of the activity yields no profit but there is just a couple of inputs, which are acquired cheaply, either because they are obtained from the government sources at an unrealistically low price, or imported, also at an artificially low price. In effect, the whole activity of the network is needed to simply distribute the initial external source of value, and the defection is clearly non-optimizing: one simply loses the stream of profit, and the only thing one is really capable of, is to participate in the process of the distribution. The network reproduces the pattern of behavior typical for the whole Soviet economy, when essentially the main activity was a "fair" redistribution of the initial source of income.

[7] In the aforementioned network, the pattern of exchange changed rather abruptly several times; at some moment, a bank, which provided *trade credit for free* appeared, then notes issued by the agents of the network reappeared; when I asked why, the answer was (literally): "Somebody decided". Clearly, there is really something or someone like a central planner, and the study of the type and volume of the media of exchange in circulation inside the network is senseless unless you have an access to the private information about the network.

where there are large monopolies, everybody has to trade with. The sector of large monopolies acts as a single actor (subject to constraints imposed by the government), and only the behavior of productive agents is the standard optimizing behavior of agents in monetary search models. The sizes of the sectors, the money supply, utility functions and production costs of agents are given exogenously. behavior described above. Trading strategies[8] prices, the amount of notes in circulation and the distribution of the money between sectors are found in an equilibrium as the result of optimizing behavior of private agents and the energy sector.

We assume that the energy sector has to please the government in order that an export constraint imposed by the government does not get worse. As a result, it has to provide the unproductive sector with an amount of energy sufficient for a production level required by the government. On the other hand, the current policy of non-acceptance of notes by UES, clearly being non-optimal from the point of view of an optimizing agent, is the result of the political will, not the result of the optimizing behavior given the structure of the economy. Hence, we model this *exogenously given* feature by introducing the lower bound on the volume of trades of the energy sector made with the money only. We also assume that the government imposes the price ceiling on the money prices of the energy sold to unproductive agents[9]. We assume that in trades inside the productive sector and between productive and unproductive agents, buyers extract all the surplus.

The prices in trades with the energy sector are determined by the latter sector in order to maximize the welfare of the sector, except for the price for the energy sold to unproductive agents in monetary form. The partial equilibrium model of similar price-discriminating behavior was constructed by Ericson and Ickes (1999).

Since the real unproductive sector is incapable to pay the necessary amount of the money as the winter crisis of non-payments and black-outs clearly demonstrated, we introduce the direct transfer of money into this sector from the government-controlled part of the energy sector. Formally, the model allows for the zero volume of the direct subsidies but the analysis of the model shows that a steady state is impossible without this transfer.

[8] E.g. for a private agent, the acceptance/non-acceptance of a note and prices she agrees to trade at in each type of meeting

[9] In order to save them; nevertheless, they remain insolvent without the direct subsidies — both in the reality and as a conclusion (not assumption!) of our model, as we will see later

Since Russia's economy and government policies are so peculiar[10], we are forced to introduce many exogenous features of the economy, which inevitably differ from standard assumptions of search-theoretical models. The additional exogenous distortions lead to very rich and interesting endogenous pattern of exchange. Hence, the study of the model requires additional efforts and leads to new results for the search-theoretical approach to Monetary Economics.

We study the dependence of the volume of notes in circulation on the sizes of the note-issuing part of the energy sector and the one which uses only money, and on the money supply, and show that the sharp decline in the use of notes can be naturally explained as the result of the increase of the money supply due to the 4-fold increase of the rouble equivalent of USD in circulation or/and the new policy of UES, both factors working in the same direction.

Our model shows that generically, the economy may be in either of two steady states with approximately the same price systems but with different volumes of notes in circulation, and hence, August 1998 crisis might have lead to the change of the steady state, the new one having less volume. We conjecture that should the productive economic activity be suppressed due to some adverse external shock, the volume of money substitutes in circulation may sharply increase.

2. MODEL SPECIFICATION

2.1. Preferences and production

Consider an economy, which consists of three sectors:

• G, the energy sector, viewed as a single actor;

• O, the sector of old low productivity firms;

[10] However, there is a rationale behind this behavior of the government. We believe that the most important reason behind the support of the unproductive sector is to preserve social stability. On the other hand, since there are instances of how the use of non-monetary means to pay taxes distorts public policy priorities (for example, the construction of Chelyabinsk subway system, as documented by Gaddy and Ickes (1998)), the government should have incentives to introduce restructuring into the old sector. Also it is clear that imposing a cash constraint, the government pursues its goals of tax collection and budget management. Modeling the incentives of the government in a macro model of the Russian economy would be an interesting and challenging issue. We leave it for future projects.

• E, the sector of high productivity firms.

The last two sectors have a continuum of agents. The total size of the population of the union of O- and E-sectors is normalized to unity, the proportions being p^O and p^E. The specification of consumption and production of private agents and non-productive agents is standard for the search-theoretical models, and the detailed description is as follows. Agents produce and consume non-storable goods (or services) at discrete points in continuous time. Agents and goods are indexed by points on a circle of circumference two. The agents have idiosyncratic tastes for goods in the sense that at any time, an agent indexed by point i has a demand for a particular variety of goods which lies within the distance $x \in (0, 1/2)$ from point i.[11] Thus we incorporate both heterogeneous consumers and differentiated goods into the economy. When an agent consumes an amount q of a demanded good, she enjoys utility $u(q)$[12]. We assume that u satisfies standard properties:

$$u \quad \text{is increasing and concave,} \tag{1}$$

and

$$u(0) < 0, \quad \text{and} \quad u'(q) \to 0 \quad \text{as } q \to +\infty. \tag{2}$$

Agent i derives no utility from any amount of good lying distance $z > x$ from i. Thus x characterizes the rate of specialization in consumption. The consumption technology of agents requires the energy. An attempt to introduce the energy as the second argument of the utility function leads to an absolutely intractable model. Instead, we introduce the technology constraint: a ratio of the average amount of energy consumed by an agent per unit of time to the average amount of other goods consumed per unit of time;[13] we denote the inverse to this ratio by κ in the model.

Each agent i can instantaneously produce good j at a fixed distance z, $2x < z < 1$, clockwise from i. Due to these properties of preferences and the production technology, agents never produce for themselves. They must trade in the exchange sector in order to consume. Agents of E-sector meet each other and agents of O-sector at random. Their trading partners arrive according to a Poisson process with the constant arrival rate α. It

[11]By a distance we mean the shortest arc between i and a given good.

[12]Lest the model becomes intractable, we assume that the utility derived by an agent from consumption of an amount q of any good within the distance x is independent of a good; we are indebted to Ken Burdett for this idea.

[13]We assume that each agent of E- and O-sectors is a part of a firm, which comprises many agents (the number of firms being large as well), so that it makes sense to speak about the average rate of consumption of this or that good by an agent.

is clear that the specification of preferences and production opportunities rules out a double coincidence of wants. Thus there can be no direct barter, and the probability of a single coincidence of wants is x.

The size of G-sector is indirectly characterized by $\alpha x Q$, the volume of energy it can produce per unit of time for sales inside the economy. The utility of consumption of G-sector is assumed to be linear:

$$U(q) = a + bq, \tag{3}$$

the motivation being as follows. The main part of the revenue for G-sector comes from the part of its output, which it sells outside the economy (i.e., abroad) since the agents of the economy cannot pay at higher world prices. This means that the additional “internal revenue” is a small addition to the fixed (from the point of view of this model) flow of “foreign revenue” R, and therefore, the contribution of the internal revenue into the utility can be calculated from the linear approximation of the utility function around R (hence, in (3), $a = \tilde{U}(R)$, where $\tilde{U}$ is “the genuine utility function”).

G-sector and agents of E-sector suffer a unit marginal cost of production, and less productive agents of O-sector suffer marginal cost $k > 1$. All agents discount future at the rate $r > 0$.

2.2. Means of exchange and trades

There is the exogenous money supply $M \in (0, 1)$. Agents may issue promissory notes (IOU's) but only those of G-sector (we call them simply notes) may circulate and be accepted by everybody; IOU's of O-sector may circulate only inside O-sector (the motivation was given above). Since the informal network in O-sector effectively acts as the central planner, O-notes are used as a simple accounting tool inside O-sector.

Both money and notes are indivisible and perfectly storable, and an agent of O-sector and E-sector can carry either 1 unit of money or note or none of those. This assumption significantly simplifies our model, and still allows us to endogenize prices as reciprocals to quantities of goods produced for a unit of money or a note. Each agent from a non-energy sector can be in one of three states: a buyer carrying a note, a buyer with a unit of money, or a seller. We denote by V_n^j, V_m^j, V_s^j, $j \in \{\mathrm{E}, \mathrm{O}\}$, the value functions of the agent in these states.

Agents of E-sector meet with each other or with agents of O-sector pairwise and at random, and in each meeting decide whether to trade, and how much to produce for a note or a unit of money. When two agents meet they cannot trade unless one agent is a seller and the other is either a buyer with a mean of exchange or the representative of the energy sector who can issue a note. Also a trade can take place if both agents are from O-sector and there is a single coincidence of wants. Evidently, the seller does not produce if she is worse off after the trade. We assume that the buyer (except for the trades with the energy sector, which has the power to act as a monopsonist, and pair-wise meetings of O-agents) makes a take-it-or-leave-it offer to the seller, which enables her to extract all the seller's surplus from trade. More precisely, we determine a quantity produced in each round of trade as a generalized Nash bargaining solution, which satisfies

$$q^j = \arg\max \left[u(q) + V_s^j - V_b^j\right]^\theta \left[V_b^j - kq - V_s^j\right]^{1-\theta},$$

where $V_b^j \in \{V_n^j, V_m^j\}, j \in \{\mathrm{E}, \mathrm{O}\}$, and θ is the bargaining power of a buyer. Due to the assumption above, buyers have absolute bargaining power in this model, i.e. $\theta = 1$. This also means that if a trade takes place, the seller produces her reservation quantity, i.e., the quantity that makes her indifferent between producing and not producing. Therefore an E-seller produces amount q_n^E (for a note) or q_m^E (for a unit of money) given by

$$V_n^E - V_s^E = q_n^E, \text{ and } V_m^E - V_s^E = q_m^E. \tag{4}$$

For an O-seller, we derive in a similar way

$$V_n^O - V_s^O = kq_n^O, \text{ and } V_m^O - V_s^O = kq_m^O. \tag{5}$$

An E-buyer decides whether to spend her means of exchange given the amount of good the seller agrees to produce. We assume that the buyer trades if she is not worse off after the trade, therefore she spends her unit of money with probability x_m^{EE} when she meets an E-producer, where

$$x_m^{EE} = \begin{cases} x & \text{iff } u(q_m^E) \geq V_m^E - V_s^E \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Similarly, an E-note holder spends her note with probability x_n^{EE} in a meeting with an E-seller, and

$$x_n^{EE} = \begin{cases} x & \text{iff } u(q_n^E) \geq V_n^E - V_s^E \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Notice that (4), (6) and (7) together imply that

$$x_m^{EE} = \begin{cases} x & \text{iff } u(q_m^E) \geq q_m^E \\ 0 & \text{otherwise;} \end{cases} \tag{8}$$

and

$$x_n^{EE} = \begin{cases} x & \text{iff } u(q_n^E) \geq q_n^E \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Cases when a buyer from the productive sector meets a seller from O-sector or vice versa can be treated in the same way. Notice that E- and O-agents do not have to make decisions whether to purchase the energy or not — they have to do it due to the consumption constraint specified earlier, and this gives E-sector the monopsony power. Due to the existence of the informal networks, the agents of O-sector do not have to search for each other, and the production level inside the sector is determined by the central planner of the sector in order to maximize the overall welfare of the sector. The agents of the sector are unproductive and cannot acquire as much energy as they used to under the former command system when the energy was extremely cheap. In other words, the energy constraint, which stems from the consumption specification, is binding.

The behavior of the energy sector is more complicated because on one hand, it optimizes its own welfare, and on the other hand, it has to satisfy the requirements imposed by the government. G sector sells its output for notes it issues and for money. The quantities of good it redeems for a note, q_n^{GE} and q_n^{GO}, for a note-holder of E-sector and O-sector, respectively, as well as the quantities bought for a unit of money are determined by the energy sector in order to maximize its welfare. G-sector also sets the price (the reciprocal of q_m^{GE}) when it sells the energy to the productive agents for money. The amount of the energy sold for a unit of money to the buyer of the unproductive sector is fixed by the price ceiling set by the government. In addition, the G-sector has to decide which mean of exchange to use in trades with E- and O- producers in order to maximize its welfare and to meet an obligation (the cash constraint imposed by the government) to sell a certain proportion, β, or more, of its output for money. The decisions fix the flows of money and notes G-sector uses in trades with each sector, n^{GO}, m^{GO}, n^{GE}, m^{GE}. When solving the optimization problem, G-sector must also ensure that the production level inside the unproductive sector is not less than αxW, the volume desired by the government. Finally, G-sector is aware that should the unproductive agents have no money to pay for the energy, the government will intervene and save the unproductive sector by forcing the money transfer from the energy sector to the unproductive sector. The transfer is made to sellers of the latter. We assume that the government sets the money subsidy as $M_s = \gamma m^{OG}$ and view the coefficient $\gamma \in [0, 1]$ as the policy instrument.

Notice that we take the behavior of the government as given, that is we do not model the government as an optimizing agent here.

2.3. Equilibrium concept

In this subsection, we specify the objectives of the agents in more details and define the equilibrium. Let $\rho = r/\alpha$ be a normalized discount rate. Let p_s^E, p_m^E, p_n^E, and p^E be the proportions of type-E sellers, money-holders, note-holders, and all E-agents, respectively, and V_s^E, V_n^E, V_m^E and

$$V^E = (p_s^E V_s^E + p_m^E V_m^E + p_n^E V_n^E)/p^E \tag{10}$$

value functions, and the weighted value function of E-agents, respectively. Introduce the flows m^{GE} and m^{EG} of money into E-sector from G-sector and in the opposite direction, and the flows of notes n^{GE} and n^{EG} into E-sector from G-sector and into G-sector from E-sector. The flows of energy consumed by E- and O-sectors are denoted by $\alpha x Q^{GE}$ and $\alpha x Q^{GO}$.

Let

$$P_m^E = p_m^E(P^E + P^O),\ P_n^E = p_n^E(P^E + P^O),\ P_s^E = p_s^E(P^E + P^O), \tag{11}$$

be masses of type-E money-holders, note-holders and sellers, respectively. Consider an E-buyer with a unit of money. Her value function satisfies the following Bellman's equation:

$$\begin{aligned} \rho V_m^E &= \frac{x m^{EG}}{P_m^E}[V_s^E + u(q_m^{GE}) - V_m^E] + x_m^{EE} p_s^E [V_s^E + \\ &+ u(q_m^E) - V_m^E] + x_m^{EO} p_s^O [V_s^E + u(q_m^O) - V_m^E]. \end{aligned}$$

In words, the expected discounted flow of value of the buyer equals the sum of all expected net gains from trade. Namely, with probability $x_m^{EE} p_s^E$ the buyer meets a seller of her type whose good she likes, or with probability $x_m^{EO} p_s^O$ the buyer meets an O-producer whose good she desires; also she has to purchase the energy from the G-sector with probability m^{EG}/P_m^E to satisfy the consumption technology constraint. As a result of all of such meetings, the buyer becomes a seller.

By similar reasoning, we can obtain the rest of Bellman's equations for E-agents.

$$\begin{aligned} \rho V_n^E &= \frac{x n^{EG}}{P_n^E}[V_s^E + u(q_n^{GE}) - V_n^E] + x_n^{EE} p_s^E [V_s^E + \\ &+ u(q_n^E) - V_n^E] + x_n^{EO} p_s^O [V_s^E + u(q_n^O) - V_n^E], \\ \rho V_s^E &= \frac{x m^{GE}}{P_s^E}[V_m^E - V_s^E - q_m^E] + \frac{n^{GE}}{P_s^E}[V_n^E - V_s^E - q_n^E] + \\ &+ x_m^{EE} p_m^E [V_m^E - V_s^E - q_m^E] + x_n^{EE} p_n^E [V_n^E - V_s^E - q_n^E] \\ &+ x_m^{OE} p_m^O [V_m^E - V_s^E - q_m^E] + x_n^{OE} p_n^O [V_n^E - V_s^E - q_n^E]. \end{aligned}$$

Let M_s be the volume of direct subsidies, which the government transfers from the energy sector to the old sector, when the latter is unable to pay for the energy. Reasoning in the same way as above, we can derive Bellman's equations for O-agents:

$$\begin{aligned}
\rho V_m^O &= \frac{xm^{OG}}{P_m^O}[V_s^O + u(q_m^{GO}) - V_m^O] + x_m^{OE} p_s^E [V_s^O + \\
&+ u(q_m^O) - V_m^O] + xp^O B, \\
\rho V_n^O &= \frac{xn^{OG}}{P_n^O}[V_s^O + u(q_n^{GO}) - V_n^O] + x_n^{OE} p_s^E [V_s^O + \\
&+ u(q_n^E) - V_n^O] + xp^O B, \\
\rho V_s^O &= \frac{xm^{GO}}{P_s^O}[V_m^O - V_s^O - kq_m^O] + \frac{xn^{GO}}{P_s^O}[V_n^O - V_s^O - kq_n^O] + \\
&+ x_m^{EO} p_m^E [V_m^O - V_s^O - kq_m^O] + x_n^{EO} p_n^E [V_n^O - V_s^O - kq_n^O] \\
&+ xp^O B + \rho x \frac{M_s}{P_s^O}[V_m^O - V_s^O].
\end{aligned}$$

Here B is the benefit from *quid pro quo* exchange between two agents of O-sector, which we do not identify explicitly in his model. As compared to the E-sellers, O-sellers have an additional value which equals the expected government direct subsidy.

The energy sector maximizes its welfare, which depends on the volumes of goods being traded with productive and unproductive agents of the economy. As we have mentioned it earlier, the main part of income of the energy sector comes from its sales abroad therefore the sector has to obey all government constraints in order to have an opportunity to earn the foreign revenue. Thus the G-sector optimizes

$$\begin{aligned}
W^G &= m^{GE} b q_m^E - m^{EG} q_m^{GE} + n^{GE} b q_n^E - n^{EG} q_n^{GE} + \\
&+ m^{GO} b q_m^O - m^{OG} q_m^{GO} + n^{GO} b q_n^O - n^{OG} q_n^{GO} - M_s
\end{aligned} \tag{12}$$

subject to the cash constraints

$$Q_n^{GO} \leq \frac{1-\beta}{\beta} Q_m^{GO} \text{ and } Q_n^{GE} \leq \frac{1-\beta}{\beta} Q_m^{GE} \tag{13}$$

and the production constraint, which ensures that the volume produced by O-sector per unit of time is not less than $\alpha x W$, which gives

$$m^{GO} q_m^O + Q^O + n^{GO} q_n^{OG} \geq W. \tag{14}$$

To summarize, we take as given the government behavior (i.e., the production and cash constraints, the policy variable γ, the supply of money provided by the government, and the price ceiling), the consumption technology requiring energy, the market power of the energy sector and the sizes of productive and unproductive sectors. The agents' preferences and production technologies as well as the amount of production feasible for the energy sector (which characterizes the size of this sector indirectly) are also given exogenously. Now we are in a position to define equilibrium as a list of endogenous variables $\mathbf{g} = \{q_m^E, q_m^O, q_m^{GE}, q_n^E, q_n^O, q_n^{GE}, q_n^{GO}, V_s^E, V_s^O, V_n^E, V_n^O, V_m^E, V_m^O, W^G, m^{GE}, m^{EG}, n^{EG}, n^{GE}, m^{GO}, m^{OG}, n^{GO}, n^{GO}, x_m^{EE}, x_m^{EO}, x_m^{OE}, x_n^{EE}, x_n^{EO}, x_n^{OE}\}$ such that

- value functions satisfy Bellman's equations
- production levels satisfy (4)– (5)
- trading strategies satisfy optimality conditions (8)– (9)
- G-sector maximizes W^G subject to (13) and (14)
- flows of liquidity satisfy steady state conditions (see below).

Notice, that unlike in our previous model (see Boyarchenko and Levendorskiǐ 2000, 2001), we do not model incentives of agents here. In fact, if the G-sector defects on redeeming a note, then eventually other agents will stop accepting notes from this sector. So as long as the sector finds it optimal to issue notes, it will not defect on them. We will show later that the existence of notes is the main vehicle for the transfer of value from E- to O-sector. O-agents may have incentives to deviate from the pattern of behavior prescribed by the central planner, but since in many cases the informal networks use non-economic means to support the participation of agents, modeling of incentive compatibility constraints for the O-agents is out of scope of our project.

3. GENERAL ANALYSIS

It is obvious, that the list of possible combinations of patterns of exchange between agents of the economy is huge, and the model may have multiple equilibria. In this project, we are going to look at a certain type of equilibrium. Namely, we will consider an equilibrium when E- and O-agents do not trade with each other. At the same time, E-agents use both media of exchange in trades with each other and the energy sector. In Boyarchenko (2000), Boyarchenko and Levendorskiǐ (2000, 2001), it was shown that such equilibrium was possible in particular, when a parameter

$h := r/x\alpha$, which we call the trading friction, is sufficiently small. This happens when the number of single coincidence meetings per unit of time is large so that agents can be patient and wait for a better trading opportunity to arrive. An E-buyer will not be willing to trade with an O-producer unless the latter produces sufficiently high quantity of good. But if the difference in productivity of agents is substantial, that is, $k-1$ is not small, the less productive agents cannot produce the amount E-buyers demand for any mean of exchange since later they cannot get for it an amount of good, which compensates for the disutility of production.

Thus, we are going to examine the case when equilibrium prices are such that it is not optimal for more productive agents to purchase goods manufactured by low productivity agents. If it had not been due to the government constraints, the energy sector would not have been buying from the O-sector as well. In fact, if the government subsidizes the latter sector heavily, then as we have shown in our interim report on the project, the G-sector does not buy from O-sector with money, so that $m^{GO} = 0$, and $m^{OG} = M_s$. In one of the following subsections, we will consider other variants of the government subsidizing policy.

As the rigorous analysis in Boyarchenko and Levendorskiĭ (2000, 2001) shows, less productive agents are willing to buy from more productive agents. In the model under consideration, neither the government nor the energy sector are willing to provide the O-sector with extra liquidity, so that the production constraint for this sector is binding, and the amount of liquidity accumulated by the sector just suffices to pay for the energy. Thus, there arise two circles of exchange, each involving G-sector. We will consider them in detail in the Subsections 4.2 and 4.3, and now we derive the steady state conditions for flow of money and notes.

In equilibrium, the flows of money from the energy sector to E-sector and in the opposite direction are equal, therefore $m := m^{GE} = m^{EG}$. By the same token, the flows of notes between the sectors satisfy $n := n^{GE} = n^{EG}$ and $\tilde{n} := n^{GO} = n^{OG}$. The money flow, m^{OG} from the old sector to the G-sector equals the money flow in the opposite direction plus the subsidy, i.e.,

$$m^{OG} = m^{GO} + M_s. \tag{15}$$

Next, we derive simplified Bellman's equations for the equilibrium under consideration. First, we notice that since E- and O-agents do not trade with each other, probabilities $x_m^{EO} = x_m^{OE} = x_n^{EO} = x_n^{OE} = 0$. On the other hand, since E-agents do trade among themselves, $x_m^{EE} = x_n^{EE} = x$, that is the agents are always willing to buy a good whenever they derive a positive utility of consumption. Next, we take into account (4)– (5).

Finally, since the energy sector acts as a monopsonist in all trades with E-sector and in note-trades with O-sector, it extracts all buyers' surplus in these trades, therefore we have

$$\begin{aligned} u(q_m^{GE}) &= V_m^E - V_s^E = q_m^E, & (16)\\ u(q_n^{GE}) &= V_n^E - V_s^E = q_n^E, & (17)\\ u(q_n^{GO}) &= V_n^O - V_s^O = kq_n^O. & (18)\end{aligned}$$

Now we can write the simplified Bellman's equations for E-sector as

$$\begin{aligned} hq_m^E &= p_s^E[u(q_m^E) - q_m^E], & (19)\\ hq_n^E &= p_s^E[u(q_n^E) - q_n^E], & (20)\\ V_s^E &= 0. & (21)\end{aligned}$$

The simplified Bellman's equations for the O-sector can be derived in a similar fashion. We can also simplify the G-sector's objective function as

$$\begin{aligned} W^G &= m(bq_m^E - q_m^{GE}) + n(bq_n^E - q_n^{GE}) + & (22)\\ &+ m^{GO}bq^O - m^{OG}q_m^{GO} + \tilde{n}(bq_n^O - q_n^{GO}) - M_s. \end{aligned}$$

4. MAIN RESULTS

We present detailed analysis and the solution to our model in Subsections 4.2 and 4.3. Before doing this, we find it sensible to present the list of technical results and the main theorem. We start with the following lemma.

Lemma 1. (i) *When redeeming a note for an unproductive agent, G-sector produces the amount* q_n^{GO}*, which solves the problem*

$$u(q)/q \to \max, \tag{23}$$

that is, the solution to the equation

$$u'(q)q = u(q), \tag{24}$$

and it charges for a note it issues to the same agents

$$q_n^{OG} = u(q_n^{GO}) > q_n^{GO};$$

(ii) q_n^{GO} *is less than the socially optimal value* q_* *defined from*

$$u'(q_*) = 1,$$

but for q_n^O*, the answer is ambiguous*;

(iii) G-sector charges for a note it issues to a productive agent the maximum amount q_n^E*, which solves an equation*

$$(1 + h/p_s^E)q = u(q), \tag{25}$$

where p_s^E *is the proportion of sellers of E-sector*;

(iv) G-sector buys with the unit of money the same amount from E-agent as when it issues a note;

(v) if the trading friction is small then q_n^E *is larger than the socially optimal amount* q_**, and in all cases,*

$$q_n^O < q_n^E < q^*,$$

where q^* *is the larger solution to the equation*

$$u(q) = q;$$

(vi) quantities G-sector produces are largest solutions to the equations

$$u(q_n^{GO}) = q_n^O; \quad u(q_n^{GE}) = q_n^E; \quad u(q_m^{GE}) = q_m^E;$$

in particular, G-sector extracts all the surplus from the trades except for the sales it makes for money to unproductive agents, when it has to sell at a loss for a fixed price;

(vii) if the trading friction is small, the quantities above are related as follows:

$$q_n^{GO} < q_n^O < q_n^{GE} = q_m^{GE} < q_n^E = q_m^E < q^*; \tag{26}$$

(viii) the ratio of the volume of notes issued to unproductive agents and the production level W required by the government is given by

$$\frac{n^{GO}}{W} = \frac{k(1-\beta)}{u(q_n^{GO})(1-\beta+\kappa)}; \tag{27}$$

thus, the volume of notes grows with W, and decreases when the volume of payments to be made with the money increases;

(ix) the ratio of the flow of energy into O-sector and the required production level is given by

$$\frac{Q^{GO}}{W} = \frac{kq_n^{GO}}{u(q_n^{GO})(1-\beta+\kappa)}; \tag{28}$$

the flow of energy into O-sector increases with both W and β;

(x) if $Q_E := Q - Q^{GO}$ and $M_E := M - m^{OG} - m^{GO}$ are positive, then the trades between G and E sector are possible, the ones with money including, and

- *the money flow from G to E and in the opposite direction is given by*

$$m = \frac{\beta Q_E}{q^G}; \tag{29}$$

- *the flow of notes from G to E and in the opposite direction is given by*

$$n = \frac{(1-\beta)Q_E}{q^G}; \tag{30}$$

- *the proportion of E-sellers, p_s^E, is a solution to*

$$(p^E - p_s^E)p_s^E = \frac{\kappa Q_E}{(P^E + P^O)q^E}, \tag{31}$$

to be more precise, a pair $q^E := q_n^E = q_m^E$ and p_s^E solve the system (25), (31);

- *the number of E-note-holders is*

$$P_n^E = P^E - (P^O + P^E)p_s^E - M + m^{OG} + m^{GO} + m. \tag{32}$$

The lemma gives the equilibrium values of all the endogenous variables as functions of the exogenous ones, and describes price discriminations. In the following theorem, we single out the main results.

Theorem 1. *Suppose that the trading friction is sufficiently small. Then the following statements hold:*

a) Generically, the economy can be in 4 steady states, which are labeled by solutions to the system (25), (31).

b) The energy sector gains in all non-monetary trades. To be more specific, in such trades, the energy sector exercises its monopoly power: this can be *seen from the fact that it produces less redeeming a note, than it gets issuing a note.*

c) *The prices in the sector of non-productive firms and in non-monetary trades between the energy sector and non-productive agents are higher than in the sector of productive agents and in the trades between the energy sector and productive agents.*

d) *In order to satisfy the government requirement, the energy sector ensures the production in the sector of non-productive agents but on the level below the socially optimal one.*

Remark. Notice that one of the primary goals of any monetary model is to determine endogenously prices of goods traded in the market. Our model provides such prices as reciprocals to the quantities of goods traded for a unit of money or a note. Moreover, the solution to our model indicates price distortions which result from the existence of money substitutes. Point *b*) above summarizes this.

4.1. On the dynamics of the notes in circulation after August 1998 crisis

From (27) and (30), we see that the volumes trades with notes of G-sector decreases with the growth of the size of G-sector, which does not accept notes; the effect on the number of notes in E-sector is not so apparent but numerical results show that it exists. The effect of the increase of the money supply M is the most prominent: see (32).

But the most interesting is the following possibility: if the trading friction is sufficiently small, then generically, the system (25), (31) has 4 solutions, the set of solutions being of the form

$$(q^E_-, p^E_{s,-}),\ (q^E_-, p^E_{s,+}),\ (q^E_+, p^E_{s,-}),\ (q^E_+, p^E_{s,+})$$

(modulo small errors). By exercising its monopoly power, *for a given proportion of E-sellers*, G-sector will choose the price system with the larger q^E, which will give it higher trade surplus. If the trading friction is small, the price system is almost the same for the both values of p^E_s (see (25)), and all characteristics of the economy are essentially the same for both possible values of p^E_s, *except for the number of E-note-holders in E-sector*, and hence, *except for the volume of non-monetary trades in E-sector*.

Thus, if due to some external shock, the number of active producers looking for buyers (in this model, the producers are sellers) increases, it is possible that the system will move into another steady state, with *a smaller volume of notes in circulation.*

4.2. Production and trades in *O*-sector and between *O*- and *G*-sectors

Let $\alpha x Q_m^{GO}$ and $\alpha x Q_n^{GO}$ be the amount of energy produced by G-sector for O-sector, for money and notes, respectively, and $\alpha x Q^{GO}$ the total amount per unit of time. Since O-agents consume goods of O-sector (an amount $\alpha x Q^O$, where $Q^O = P^O q^O$, and $\alpha x q^O$ is an amount produced by O-agent per unit of time for other O-agents), goods of E-sector purchased for notes, and energy (an amount $\alpha x(Q_m^{GO} + Q_n^{GO})$), the consumption technology constraint assumes the form

$$Q^O = \kappa(Q_m^{GO} + Q_n^{GO}). \tag{33}$$

If there exists the price ceiling for sales to O-sector made in the monetary form, that is, q_m^{GO} is fixed, then the money flow $\alpha x m^{OG}$ from O-sector to G-sector is determined by

$$m^{OG} = Q_m^{GO}/q_m^{GO}, \tag{34}$$

and the money flow in the opposite direction satisfies the steady state condition (15).

Remark. If the government subsidizing policy is state contingent, that is if the government subsidizes the O-sector only when the latter is short of money, then in view of (34) and (15), the volume of subsidy must be determined by the optimizing behavior of G-sector, and this behavior depends on the constraints imposed by the government. This gives the possibility of studying the volume of *direct subsidies*, which the government will have to arrange for, as a function of its own policies of *indirect subsidies* and restrictions; should the value function for the government be specified, it gives an opportunity to *endogenize the government policies*. We leave this possibility for future research.

While writing (15), we implicitly assume that the government closely monitors the subsidized O-sector and does not allow O-agents to use subsidies except for buying the energy, which is necessary for production of the sector. The model admits a modification, when due to imperfect monitoring, a part of subsidies is spent to buy goods of E-sector (due to the informal connections, O-sector does not need money for trades inside the sector: it can use either direct barter or money substitutes its agents issue).

Similarly, G-sector has no incentive to provide O-sector with additional liquidity, which will be spent not to boost the production inside O-sector (which is the concern of G-sector because of the production restriction imposed by the government) but to simply derive additional utility from consumption of goods produced by E-sector.

Recall that the G-sector has to satisfy the production constraint, (14), and the sales constraint, (13), imposed by the government. Due to the price ceiling established by the government, it is not profitable for the G-sector to sell the energy to the O-sector for money, hence, the constraint (13) is binding:

$$Q_n^{GO} = \frac{1-\beta}{\beta} Q_m^{GO}. \tag{35}$$

We can also safely presume that the constraint (14) is binding as well: be it not the case, there would be no need for the government intervention at all. Hence, we can replace (14) with

$$m^{GO} q_m^O + Q^O + n^{GO} q_n^O = W. \tag{36}$$

Due to (35) and (36), G-sector simultaneously minimizes Q_m^{GO} and Q_n^{GO}.

Let the government subsidy $M_s = \gamma m^{OG}$, $\gamma \in [0,1]$. If $\gamma = 1$, that is the government subsidizes the old sector heavily, then by excluding Q^O, Q_m^{GO} and Q_n^{GO} from (33), (36) and (35), we obtain

$$n^{GO} q_n^O = W - \kappa \left[1 + \frac{\beta}{1-\beta}\right] n^{GO} q_n^{GO},$$

which can be transformed first into

$$n^{GO}\left(q z_n^O + \frac{\kappa}{1-\beta} q_n^{GO}\right) = W,$$

and then into

$$(Q_n^{GO} :=) n^{GO} q_n^{GO} = \frac{W}{\kappa/(1-\beta) + q_n^O/q_n^{GO}}. \tag{37}$$

On the other hand, if $\gamma = 0$, i.e., the government does not subsidize the old sector at all, then

$$m^{GO} = m^{OG} = \frac{\beta Q_n^{GO}}{(1-\beta) q_m^{GO}},$$

and from the corresponding Bellman's equation, we obtain

$$\begin{aligned} hkq_m^O &= \frac{m^{OG}}{P_m^O}[u(q_m^{GO}) - kq_m^O] \\ hkq_m^O &= P^O[u(q_m^{GO}) - kq_m^O]. \end{aligned}$$

By solving the last equation for q_m^O, we arrive at

$$q_m^O = \frac{u(q_m^{GO})}{k(h/P^O+1)}.$$

Now, we can substitute for m^{GE} and q_m^O in (36) to get

$$\frac{\beta Q_n^{GO}}{(1-\beta)q_m^{GO}} \frac{u(q_m^{GO})}{k(h/P^O+1)} + n^{GO}\left(qz_n^O + \frac{\kappa}{1-\beta}q_n^{GO}\right) = W,$$

which can be transformed into

$$Q_n^{GO} = W\left(\frac{\beta u(q_m^{GO})}{k(1-\beta)q_m^{GO}(h/P^O+1)} + \frac{\kappa}{1-\beta} + \frac{q_n^O}{q_n^{GO}}\right)^{-1}. \qquad (38)$$

Since W, κ, q_m^{GO}, and β are fixed, and both in (37) and (38) the LHS is the quantity to be minimized, we see that G-sector maximizes q_n^O/q_n^{GO}. From (18), we know that $q_n^O = u(q_n^{GO})$, therefore the optimization problem is:

$$\frac{u(q_n^{GO})}{q_n^{GO}} \to \max. \qquad (39)$$

If $\gamma \in (0,1)$, then the corresponding Bellman's equation implies

$$\begin{aligned} hkq_m^O &= P^O[u(q_m^{GO}) - kq_m^O] - \frac{\gamma m^{OG}}{P_s^O}kq_m^O \text{ or} \\ q_m^O &= \frac{u(q_m^{GO})}{k(1+h/P^O+\gamma m^{OG}/P^O/P_s^O)}. \qquad (40) \end{aligned}$$

Substituting the last equation into (36), we arrive at

$$\begin{aligned} (1-\gamma)m^{OG}\frac{u(q_m^{GO})}{k(1+h/P^O+\gamma m^{OG}/P^O/P_s^O)} &+ \qquad (41) \\ +n^{GO}\left(qz_n^O + \frac{\kappa}{1-\beta}q_n^{GO}\right) &= W \\ m^{OG} = \frac{\beta Q_n^{GO}}{(1-\beta)q_m^{GO}}&. \qquad (42) \end{aligned}$$

It follows from (41) and (42) that Q_n^{GO} is a solution to a quadratic equation. Tedious computations show that the smaller root of this equation decreases in q_n^O/q_n^{GO}, so the optimization problem remains the same.

Under conditions (1)– (2), the solution to the optimization problem (39) exists and it is unique, and $q^{GO}_{n,\mathrm{opt}}$, the optimal amount of energy produced for a note for agents of O-sectors, is the unique solution to the equation

$$u'(q)q = u(q). \tag{43}$$

When $q^{GO}_{n,\mathrm{opt}}$ is found, we can find step by step:

• the amount O-agent produces for a note:

$$q^{OG}_n = u(q^{GO}_{n,\mathrm{opt}})/k; \tag{44}$$

• the flow of notes from G-sector into O-sector, and the flow of notes into opposite direction

$$n^{GO} = n^{OG} = \frac{W(1-\beta)}{q^{OG}_n(1-\beta+\kappa)}; \tag{45}$$

• the flow of energy to O-sector, produced for notes

$$Q^{GO}_n = n^{GO} q^{GO}_n; \tag{46}$$

• the flow of energy into O-sector, produced for money

$$Q^{GO}_m = \frac{\beta}{1-\beta} Q^{GO}_n; \tag{47}$$

• the flow of money into O-sector

$$m^{GO} + M_s = \frac{Q^{GO}_m}{q^{GO}_m}; \tag{48}$$

• the total flow of energy into O-sector

$$Q^{GO} = \frac{Q^{GO}_n}{1-\beta}; \tag{49}$$

• the amount O agent produces for a unit of money from (40);

• the production by O-sector for G-sector

$$Q^{OG} = n^{GO} q^{O}_n + m^{GO} q^{O}_m; \tag{50}$$

• the production flow inside O-sector

$$Q^O = W - Q^{OG}; \tag{51}$$

• the disutility flow suffered by G sector due to trades with O-sector

$$U_{G,O} := b[n^{GO} q_n^O + m^{GO} q_m^O - W]; \tag{52}$$

• the utility extracted by O-sector from G-sector

$$U_{O,G} := P^O u(Q^O/P^O) + m^{OG} u(q_m^{GO}). \tag{53}$$

4.3. Production and trades in *E*-sector and between *E*- and *G*-sectors

We assume that after G-sector has produced the optimal (given the government restrictions) amount of energy Q^{GO} given by (49), the remainder $Q_E := Q - Q^{GO}$ is positive.

The consumption technology constraint (the constant ratio of the volume of consumption goods from non-energy sector to the volume of goods from the energy sector) is

$$\frac{p_m^E p_s^E q_m^E + p_n^E p_s^E q_n^E}{p_m^E (m/P_m^E) q_m^{GE} + p_n^E (n/P_n^E) q_n^{GE}} = \kappa, \tag{54}$$

and the sales constraint is

$$m q_m^{GE} + n q_n^{GE} \le Q_E. \tag{55}$$

Finally, the volume-in-monetary-form constraint is

$$m q_m^{GE} \ge \frac{\beta}{1-\beta} n q_n^{GE}. \tag{56}$$

Given the restrictions listed above and the standard assumption that nobody is worse off after a trade, G-sector solves an optimization problem

$$m[b q_m^E - q_m^{EG}] + n[b q_n^E - q_n^{GE}] \to \max, \tag{57}$$

where b is the coefficient in (3).

From (16)– (17) by using properties (1)– (2) of the utility function u, we find that $q_m^{GE} < q_m^E$ and $q_n^{GE} < q_n^E$, if G-sector extracts all the buyers' surplus from agents of E-sector. We conclude that if the coefficient b in (57) is greater than or equal to 1, then it is optimal for G-sector to trade with E-sector, on conditions which do not make E-sector worse off. It is seen easily from (19) and (20), that q_m^E and q_n^E solve the same equation, that is E-agents produce the same amount of good, q^E, in trades of all types, and q^E is a solution to the equation

$$(1 + h/p_s^E)q = u(q). \tag{58}$$

Now (16) and (17) imply that G-sector produces both for a unit of money and for a note the same amount of energy, q^G, which solves the equation

$$u(q^G) = q^E. \tag{59}$$

Under condition (2), (58) may have two solutions, one solution or none; if $u(0) < 0$ is sufficiently close to 0, there exist two of them, and it is easy to see that the difference $bq^E - q^G$, which G-sector maximizes, is larger for the larger q^E (we assume that $b \geq 1$). Since the G-sector can dictate prices in relations with E-sector, she will choose greater q^E (and hence, greater q^G).

Since $q_m^E = q_n^E = q^E$, $q_m^{GE} = q_n^{GE} = q^G$, and $p_m^E + p_n^E = p^E - p_s^E$, we can rewrite (54)– (57) as

$$(p^E - p_s^E)p_s^E = \frac{\kappa q^G}{\big(P^E + P^O\big)q^E}\,[m+n], \tag{60}$$

$$m + n \leq \frac{Q_E}{q^G}, \tag{61}$$

$$m \geq \frac{\beta}{1-\beta}\,n, \tag{62}$$

$$[m+n](bq^E - q^G) \to \max. \tag{63}$$

It follows that the constraint (61) is binding, and the total flow of liquidity $m + n^{EG}$, which flows from E-sector into G-sector, is determined by

$$m + n = \frac{Q_E}{q^G}. \tag{64}$$

By substituting (64) into (60), we obtain an equation for p_s^E:

$$(p^E - p_s^E)p_s^E = \frac{\kappa Q_E}{(P^E + P^O)q^E}, \tag{65}$$

Suppose that E-sector is not very small, so that the total flow of liquidity into it, determined from (64), is less than the total mass of E-sector:

$$Q_E/q^E < P^E. \tag{66}$$

Then from the system (58), (65), we can find a pair (q^E, p_s^E), which is independent of a choice of m satisfying

$$\frac{Q_E\beta}{q^G} \leq m < M_E/2. \tag{67}$$

Notice that G-sector maximizes

$$(m+n)[bq^E - q^G] = (m+n)[bq^E - u^{-1}(q^E)].$$

When the trading friction is small, we conclude from (58)– (1), that

$$bq^E - u^{-1}(q^E) \uparrow \quad \text{as} \quad q^E \downarrow,$$

that is, when the price level in E-sector and in trades between E and G sectors increases, the monopolist gains more. But from (58), it is equivalent to $p_s^E \downarrow$; the only way G-sector can decrease the size of p_s^E, is to issue as many notes as possible. Thus, G-sector does not use money unless forced to, that is, the left constraint in (67) is binding:

$$m = \frac{Q_E\beta}{q^G}. \tag{68}$$

Notice that m is the number of E-money-holders who trade with G-sector at a given moment of time, and find, step by step:

- a pair (q^E, p_s^E), which solves a system (58), (65), with the greater q^E of two for the same p_s^E;
- $q^G = q^G(q^E)$;
- the number of E-note-holders who trade with G-sector:

$$n = \frac{Q_E(1-\beta)}{q^G}; \tag{69}$$

- the total number of E-money-holders:

$$m_E = M_E - m; \tag{70}$$

- the total number of E-note-holders:

$$n_E = P^E - (P^O + P^E)p_s^E - m_E; \tag{71}$$

- the weighted value function of type-E agents:

$$V^E = (1 - p_s^E/p^E)q^E. \tag{72}$$

4.4. The proof of "no-buying-for-notes-from-more-productive agents-by-less productive agents"

Suppose, that an O-seller acquires a note from an E-note-holder. If the trading friction is sufficiently small, the latter will not buy unless the former agrees to produce q^E. Hence, the former will suffer the disutility kq^E. On acquiring the note, the O-agent will buy the energy, q_n^{GO}, and derive the utility $u(q_n^{GO})$. By comparing equations for the optimal q^E, which is the larger root (we assume it exists) and for the optimal q_n^{GO}, and taking into account (1)– (2), we find that $u(q_n^{GO}) \leq q^E$. Hence, it is not individually optimal for an O-seller to produce for a note to E-note-holders.

4.5. Example

Let c, d be positive constants, and $\theta \in (0, 1)$. Then

$$u(q) = -c + dq^\theta \tag{73}$$

satisfies (1) and (2), and (43) assumes the form

$$\theta dq^\theta = -c + dq^\theta. \tag{74}$$

We find

$$q_{n,\mathrm{opt}}^{GO} = \left(\frac{c}{d(1-\theta)}\right)^{1/\theta}, \tag{75}$$

and after that all endogenous variables, which describe relations between G and O sectors. After that, we can find numerically q^E as the larger solution to (58) (which exists, if $c > 0$ is not too large); q^E is a function of p_s^E. Next, we find p_s^E, after that, q^G from (59), and the rest of the endogenous variables, which characterize the relations in E-sector and between E- and G-sectors.

4.6. Calibration problem

Certainly, it would be highly desirable to calibrate the model, and this was our intention when we planned the project. However, the following principal problems with the data available seem to be too difficult to overcome:

• In the statistical data, all kind of barter are treated the same whereas in the framework of our model, the goods acquired for the transaction purposes must be treated as a mean of exchange;

• To calibrate the model, we need to distinguish the volume of trades, the flows of means of exchange, etc., between non-restructured and new or restructured enterprizes, and we were unable to find reasonable proxies for these two sectors; all the data seem entangled.

5. POLICY CONCLUSIONS

1. The policy of direct and indirect subsidies ensures that the natural monopolies produce for unproductive agents but on the level below the socially optimal one.

2. If a part of the natural monopolies does not accept notes, then unproductive agents cannot survive without direct subsidies.

3. The increase of the money supply and of the size of the part of the natural monopolies which does not accept the money substitutes lead to the contraction of the volume of money substitutes in circulation.

4. The larger the production level in the unproductive sector, which the government requires, the larger the amount of the direct subsidies needed, and the larger the volume of money substitutes in circulation.

5. Reduction in direct subsidies increases the value of holding money for O-agents, which creates an incentive for them to produce for money.

6. Our model shows that generically, the economy may be in either of two steady states with approximately the same price systems but with different volumes of notes in circulation, and hence, August 1998 crisis might have lead to the change of the steady state, the new one having less volume of notes in circulation. We conjecture that should the productive economic activity be suppressed due to some adverse external shock, the volume of money substitutes in circulation may sharply increase.

7. As it was already mentioned in the paper, the role for money substitutes arises in the model endogenously. To summarize, the role of such money is two-fold: first, in trades with the productive sector, it is an additional mean of exchange in the economy with the lack of liquidity, second, and more important, the ability to issue universally accepted money substitutes allows the energy sector to extract value from the unproductive sector, and satisfy at the same time the production constraint imposed by the government. Notice, that the energy sector cannot exercise its monopoly power in monetary trades with the unproductive sector due to the price ceiling. The value extracted by the energy sector from the productive sector is

(partially) transferred in the form of direct subsidies to the unproductive sector.

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